Open Coverings of the Rationals Need Not Cover The Reals

A standard result in a Real Analysis course to show that rationals form a set of measure zero. The proof starts with any positive number epsilon, and then specifies an infinite collection of open intervals with total length less than epsilon.

What is often (always?) ignored is that this means that for small enough epsilons there must be real numbers which lie in none of the open intervals. But how can this be? This talk examines this seeming paradox.